JOURNAL
OF THE
WASHINGTON ACADEMY OF SCIENCES

VOL. 18 OCTOBER 19, 1928 No. 17

GEOPHYSICS.—*Evaporation from large bodies of water and some figures for Chesapeake Bay.*[1] ROGER C. WELLS, U. S. Geological Survey.

The subject of evaporation may be considered on several different scales. For the earth as a whole it is obvious that the total evaporation must very nearly equal the total precipitation. Both processes take place to a much greater extent in warm than in cold regions, however. In comparing large land and water areas in the same latitude it appears that in the water areas evaporation must exceed precipitation slightly, the excess moisture passing in the atmosphere to the land, whereas in most land areas precipitation is in excess, in spite of the increased surface offered by foliage and vegetation, and the excess water precipitated returns through lakes and rivers to the ocean.

Data on rainfall are much more abundant than data on evaporation, yet in theory the two processes are complementary, and even practical considerations would seem to warrant further scrutiny of the phenomena of evaporation.

Numerous formulas have been proposed for computing evaporation from the humidity of the air,[2] but when any particular humidity has been specified that near the surface of the water has generally been used. Unfortunately these formulas are of little use for computing

[1] Published by permission of the Director of the United States Geological Survey. Received August 6, 1928.

[2] G. J. LIVINGSTON, Mon. Weath. Rev. 1908 and 1909. HUMPHREYS, *Physics of the Air*, p. 249. DURYEA and HAEHL, Trans. Am. Soc. Civil Engr., 80: 1829. 1916. HORTON and GRUNSKY, *Hydrology of the Great Lakes*. Engineering Board of Review of the sanitary district of Chicago, Pt. III, Appendix II. 1927.

evaporation from most natural bodies of water, either because no weather station is located near by with available records, or, as may be the case, the humidity at the weather station is different from that at the lake or other body of water. The variation of humidity with altitude has been very little studied, so that extrapolation to distant points is rather uncertain.

Formulas of the kind referred to are nearly all modifications of the so-called Dalton equation, but it may be noted in passing that a different line of attack is offered by Cummings and Richardson[3] in an equation in which evaporation depends on the "heat budget" of a unit area of the water, but this method also must rely, apparently, on tests with pans and observations at the localities under study.

The relative humidity at the surface of a natural body of water must be very close to 100 per cent, and similarly the absolute humidity at high altitudes where the temperature falls below the freezing point must be very low. But what is the average distribution of moisture or vapor-pressure gradient between these two extremes?

The gradient is certainly not linear. It must resemble the concentration gradient of a dissolved salt in diffusion experiments. True diffusion of the water molecules through the air from the water surface will account for the rate of natural evaporation only when that may prove to be the slowest feature of the process.[4] Now moist air is lighter than dry air, so that convection currents are set up, and an entirely different rate of transfer is developed. Winds assist in causing some vertical transfer in a way that G. I. Taylor has attempted to cover by the concept of "eddy diffusion,"[5] in which the "diffusion constant" remains to be evaluated and may possibly be a function of the altitude or some other factors.

Some observations of the vapor-pressure gradient were made by Professor Bigelow at the Reno reservoir, Nevada, in 1907.[6] The data referring to tower No. 3, which was located near the middle of the reservoir, are of greatest interest. Each of his figures is an average of a week's daily observations of the partial vapor pressure of the moisture in the air, $p'(=e_d)$, at different hours of the day. In Table I are shown the averages of two weeks' observations. The figures for zero altitude show the vapor pressure of water at its temperature at

[3] Phys. Rev. **30**: 527. 1927.
[4] H. S. TAYLOR, *Physical Chemistry*, p. 933.
[5] Phil. Trans. **215**: 1. 1915.
[6] Mon. Weath. Rev. **36**: 28. 1908.

TABLE I.—VAPOR PRESSURE, p', ABOVE THE SURFACE OF RENO RESERVOIR, AUGUST 1–10, 12–17, 1907

Height	p' in mm. of mercury							Mean
	1 a.m.	5 a.m.	8 a.m.	11 a.m.	2 p.m.	5 p.m.	8 p.m.	
45 feet	5.9	6.0	7.2	7.1	6.2	6.4	5.8	
35	5.9	6.0	7.4	7.2	6.4	6.1	5.6	
25	6.2	6.1	6.6	6.7	6.9	6.1	5.9	
15	7.1	6.3	7.6	7.0	7.1	6.7	6.3	
7	7.5	6.7	7.5	7.8	8.2	7.5	6.9	
2	7.8	6.8	7.7	8.1	8.1	7.9	6.9	
0.5 inch	11.0	9.1	10.0	11.8	14.4	12.6	9.7	
0	15.3	14.6	15.2	18.6	20.3	18.1	15.2	
Diminution of vapor pressure with the altitude expressed as percentage of the difference between the pressure at 0 and 45 feet altitude								
45 feet	100	100	100	100	100	100	100	100
35	100	100	98	99	98	102	102	100
25	97	99	107	104	95	102	99	103
15	87	97	95	101	94	97	95	95
7	83	92	96	94	86	91	88	90
2	80	91	94	91	86	87	88	88
0.5 inch	46	64	65	59	42	47	59	55
0	0	0	0	0	0	0	0	0

the time of observation. As the air above the water was never saturated its temperature does not concern us here. The change of the diffusion constant with temperature is very small for water vapor.

Unfortunately figures for the wind velocities corresponding to the vapor pressures shown in Table I are not given in Bigelow's paper. He says, however, "In the forenoon it is calm until about 10 o'clock, when a breeze begins in the southeast, increasing in strength up to 30 or 40 kilometers per hour on many afternoons." The average velocity at the surface of the water for all winds ranging between 20 and 40 kilometers is given as 28 kilometers for the period 2 to 5 p.m. The actual velocity of the wind for any particular period is not stated.

The influence of the wind can perhaps be seen in the humidity. The highest absolute humidity at heights from 15 to 45 feet occurs at 8 a.m. In the afternoon, when the breeze has sprung up, the humidity at these heights is lower in spite of the fact that the humidity at the surface of the water is much higher.

As Table I and figure 1 show, the vertical vapor-pressure gradient is very steep indeed immediately above the surface of the water.

Over 50 per cent of the diminution observed at 45 feet is found in the first half inch above the water. At a height of 2 feet we have similarly 88 per cent as an average of the figures for the seven periods. The diminution at heights above 45 feet must go on slowly though doubtless with considerable irregularity. Further data obtained by Bigelow at the Salton Sea, California,[7] 7500 feet from the shore, show a vapor pressure gradient that is slightly steeper but otherwise similar to the one found at the Reno reservoir. The values plotted in figure 1 are the means of the percentages given in Table I.

For the purpose of developing a formula for evaporation in terms of the humidity of the air it would seem that the humidity at an altitude of at least 1 foot, or perhaps best that at 2 feet, should be used, on account of the more steady relation it holds both to the vapor pressure of the water and to the partial vapor pressure at higher altitudes.

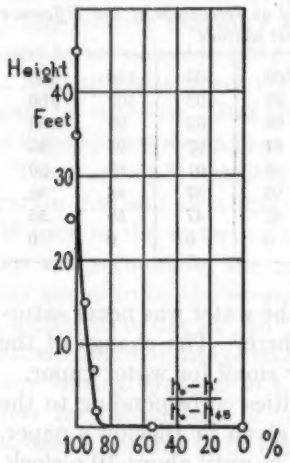

FIG. 1.—Vapor pressure-gradient, Tower No. 3, Reno Reservoir, Nevada, according to Bigelow's observations in 1907.

One form of the Dalton equation is

$$E = \frac{k}{B}(p - p')(1 + cw) \qquad (1)$$

in which E is the evaporation in centimeters per hour, B the barometer in millimeters of mercury, p the vapor pressure of the water evaporating and p' the partial pressure of moisture in the air, both in millimeters of mercury, w the velocity of the wind in kilometers per hour, and k and c constants.

The writer has derived values for k and c from some of Bigelow's data shown in Table II. Here p' is the vapor pressure of moisture in the air determined with a sling psychrometer 1 or 2 feet above the water surface. The figures are averages grouped according to different values of the velocity of the wind.

On plotting $\frac{E}{p - p'}$ against w it is seen that the values fall into three groups and are rather discordant. A graphic linear solution for

[7] Mon. Weath. Rev. 38: 310. 1910.

TABLE II.—BIGELOW'S RESULTS ON EVAPORATION FROM A 6-FOOT TANK FLOATED IN THE MIDDLE OF THE RENO RESERVOIR (TOWER 3, PAN 1)

Wind	*t Temp. of water*	*p Vap. Pr. of water*	*p' Partial Pr. of moisture in air*	*p-p'*	*E Evaporation*
Km. per hr.	*°C.*				*Cm. per hr.*
1	15.3	13.0	6.7	6.3	0.033
2	16.1	13.7	6.6	7.1	.035
2	16.7	14.3	7.0	7.3	.032
2	17.2	14.7	7.0	7.7	.037
4	19.7	17.2	7.5	9.7	.040
4	18.3	15.8	7.0	8.8	.042
6	21.3	19.0	7.3	11.7	.047
6	20.0	17.5	7.0	10.5	.047
13	17.6	15.1	7.3	7.8	.043
13	17.8	15.3	8.8	6.5	.039
14	18.0	15.5	8.7	6.8	.039
14	17.5	15.0	6.5	8.5	.046
15	18.0	15.5	6.5	9.0	.055
16	19.2	16.7	6.7	10.0	.051
17	21.2	18.9	7.8	11.1	.061
17	20.2	17.8	7.3	10.5	.059
21	17.0	14.5	5.3	9.2	.044
22	16.9	14.4	5.4	9.0	.049
22	17.6	15.1	6.6	8.5	.044
24	16.7	14.3	5.7	8.6	.052
25	18.4	15.9	6.3	9.6	.059
26	17.0	14.5	6.3	8.2	.061
27	20.2	17.8	7.1	10.7	.069
28	19.3	16.8	7.6	9.2	.068

$\frac{E}{p - p'}$ when $w = 0$ gives $\frac{k}{B} = 0.0044$, whence, as $B = 658$ for Reno, $k = 2.90$ and $c = 0.018$, or $\frac{k}{B} = 0.0038$ for a barometer of 760 mm.

When equation (1) was applied with these constants to test some evaporation records for 6-foot pans sunk in the ground at several localities[8] the calculated results were generally too high for the higher temperatures and too low for the lower temperatures, with wide individual variations, ranging from 77 to 217 per cent of the observed evaporation. It is clear either that the Dalton equation is of little value or that some factors of these records with pans have not been correctly determined. On the assumption that the latter alternative may be the correct one an attempt has been made to apply the equation to

[8] HORTON, Mon. Weath. Rev. 49: 553. 1921.

compute the annual evaporation from Chesapeake Bay from data available in the Weather Bureau records for Baltimore and Norfolk. Even if the result is only approximate, it may serve to induce further study of the problem, or calculations by others by other methods. The steps of procedure are as follows.

We have the mean monthly temperature and humidity for 37 years at Baltimore and Norfolk. These figures are first interpolated slightly in order to obtain figures corresponding to the upper half and the lower half of the bay, by adding 28 per cent of their difference to the Baltimore readings and subtracting 21 per cent of their difference from the Norfolk readings. The water and air temperatures were considered to be the same. The vapor pressure of water at each temperature is corrected very slightly, only 0.7 and 1.0 per cent for the upper half and lower half of the bay, respectively, for the salinity of the water, giving $p_{Cor.}$ The instruments at Baltimore and Norfolk are about 100 feet and 170 feet respectively above the level of the bay, so that the interpolated humidity of these heights, p_h, should be increased to correspond with an altitude of 1 or 2 feet above the water. It is difficult to decide just how great this increase should be. On the basis of the curve in figure 1, and allowing slightly for the greater altitude of the instruments a figure of 87 per cent of the difference $p_{Cor.} - p_h$ has been used to reduce the available data on humidity to an altitude of 2 feet above the water. This gives the figures in Table III.

TABLE III.—HUMIDITY

Month	Observed		Computed for 2 feet above water	
	Baltimore	Norfolk	Upper half of bay	Lower half of bay
January	71	75	75	77
February	68	75	74	77
March	67	74	72	77
April	62	72	70	74
May	65	74	72	75
June	68	77	75	79
July	69	79	76	80
August	72	80	77	81
September	73	80	78	81
October	71	78	76	80
November	69	74	74	76
December	70	75	74	76

TABLE IV.—EVAPORATION FROM CHESAPEAKE BAY

	Mean surface temperature	p (cor.)	p'	p-p'	w	E	E
Northern half							
	°C.	Mm.	Mm.	Mm.	Kilometers per hr.	Cm. per hr.	Cm. for month
January	2.1	5.3	4.0	1.3	8.1	0.0057	4.3
February	3.0	5.7	4.2	1.5	8.7	.0066	4.4
March	6.6	7.2	5.3	1.9	8.0	.0082	6.1
April	12.3	10.6	7.5	3.1	9.3	.0138	9.9
May	18.3	15.7	11.4	4.3	8.0	.0186	13.9
June	22.8	20.7	15.6	5.1	7.2	.0219	15.8
July	25.4	24.1	18.4	5.7	7.0	.0244	18.4
August	24.5	22.9	17.9	5.0	6.7	.0212	15.8
September	20.9	18.4	14.5	3.9	6.3	.0164	11.8
October	15.2	12.9	9.9	3.0	7.2	.0128	9.6
November	8.8	8.4	6.3	2.1	7.4	.0089	6.4
December	3.6	5.8	4.4	1.4	7.1	.0060	4.5
							120.9
Total annual evaporation = 48 inches							
Southern half							
January	4.0	6.0	4.7	1.3	10.6	0.0059	4.4
February	5.0	6.4	5.0	1.4	10.3	.0063	4.2
March	8.3	8.1	6.3	1.8	11.3	.0082	6.1
April	13.3	11.4	8.5	2.9	11.1	.0132	9.5
May	18.8	16.1	12.3	3.8	9.3	.0169	12.6
June	23.3	21.3	16.8	4.5	8.3	.0197	14.2
July	25.2	23.8	19.2	4.6	8.3	.0201	15.0
August	25.0	23.6	19.2	4.4	8.0	.0191	14.3
September	21.6	19.2	15.8	3.4	7.5	.0146	10.5
October	16.4	13.9	11.2	2.7	9.0	.0119	8.9
November	10.2	9.2	7.1	2.1	9.1	.0093	6.7
December	5.5	6.7	5.2	1.5	9.5	.0067	5.0
							111.4
Total annual evaporation = 44 inches							

The anemometers at Baltimore and Norfolk are respectively 113 feet and 205 feet above the ground. Bigelow found a uniform rise in wind velocity over the Reno reservoir of 0.66 per cent per foot of altitude. This rate was used up to 25 feet. Above 25 feet the variation was calculated by the Stevenson formula, quoted by Humphreys, so that, as a result, the Baltimore and Norfolk wind observations were reduced by 37 per cent and 49 per cent respectively, in order to repre-

sent wind velocities at the surface of the bay. The figures given in the tables were obtained by reduction from observations for 1907, 1908, 1920, and 1921. For the earlier years the anemometer altitudes were different.

Table IV is calculated by equation (1) for the special case in which p is the vapor pressure corrected for salinity and p' is the partial vapor pressure derived for the humidities of Table III corresponding to a height of about 2 feet above the surface of the water. It is easily seen that

$$p - p' = 0.87\ (p_{Cor.} - p_h)$$

The final result in Table IV is an annual evaporation of 46 inches (which happens to be almost exactly equal to the rainfall, 45 inches). The evaporation calculated by using the humidity observed at the Weather Bureau stations directly, without any correction for altitude, is 53 inches. This is probably too large. On the other hand, if the vapor-pressure gradient for the upper part of the curve is taken to be twice as steep as that used, an improbable assumption, the result is 38 inches. These results indicate for one thing the need of further study of the vapor-pressure gradient above natural bodies of water, although, as has been stated, there may still be fundamental errors in the Dalton equation, or even in the Reno data from which the constants were derived. However, the results are of the same order as some actual measurements of evaporation from large stretches of water near Arles, France, 15 miles from the shore of the Mediterranean, with about the same mean annual temperature.[9]

Summary

Some new constants have been derived for the Dalton equation for evaporation, to be used for large bodies of water when the humidity at a point 1 or 2 feet above the water, the temperature, and the velocity of the wind can be obtained or calculated from available data. Applied to Chesapeake Bay the formula gives an annual evaporation of 46 inches.

[9] Hegly, Ingénieur en Chef, Ponts et Chaussées, Chaumont (Haute-Marne), France. Trans. Amer. Soc. Civil Eng. 80: 1994. 1916.

PHYSICS.—*Unit of thermal resistance: the "fourier."*[1] D. ROBERTS HARPER 3D, Union College, Schenectady, N. Y.

The proposal is advanced to assign to the practical metric unit of thermal resistance the name "fourier," in honor of the foremost contributor to the theory of thermal conduction. The absolute unit of energy, the erg, being too small for many purposes, the joule is regarded by the author as the practical metric unit of energy, and the watt for rate of energy transfer.

Accordingly the *fourier* is defined as that thermal resistance which will transfer heat energy at the rate of one joule per second (one watt) for each degree (Centigrade) temperature difference between its terminal surfaces.

The "laboratory fourier" may be visualized as a prism of silver or copper about 4 cm. long and 1 cm^2. cross section.

The paper outlines briefly the psychological reasons for naming some elementary units in each branch of physics, and discusses various possible choices of metric and English units for heat transfer problems.

Two tables are given; one, the author's estimate of the most suitable values to use for thermal resistivities of about 30 common materials; the other, a conversion table between 9 of the units of heat transfer per unit area, common in the literature of today.

The Ohm-Fourier law is, in its application to electrical quantities, familiar to every schoolboy who has taken a course in physics. In the application to thermal problems, it is not so generally recognized, and more or less haze surrounds the concepts involved in heat-conduction calculations. Would not the psychologist locate one of the reasons for this in a lack of definite names for the thermal units?

When a high school student first learns that "amperes are volts divided by ohms," he is generally learning words only, and has scant understanding of the fundamental concepts, but nevertheless he very soon acquires a sufficient grasp of elements to make correct calculations in a surprisingly large percentage of the problems which come to him. The familiarity gained with these three terms seems to lay a foundation that assists greatly in his more advanced studies, when developing the philosophy of electrical relations and determining the logical order in which units must be defined. Would the same result be secured if we had no term "volt," but only a phrase "work per unit quantity of electricity," or if instead of the single term "ampere" we had a dozen or so of expressions, each nameless, for rate of transfer of electric quantity? There is more to it than mere clumsiness of language. Not alone is one's vocabulary impoverished and the ease of expressing ideas hampered by lack of names; ideas themselves are by no means so clear when no name can be associated in the mental process. Every laboratory student in physics and engineering early gets a rather definite visualization of, let us say, 1000 ohms, a megohm,

[1] Received August 10, 1928.

a few microhms. Few amongst even mature workers have any such concrete mental picture of thermal resistance.

There will be a distinct advance in the understanding of the elements of problems in thermal conduction, if physicists and engineers can agree upon the use of a few primary units in which to express relations, and will give those units names. Sporadic attempts to do so have been noted in the literature. Dr. Hering's proposal for a temporary use of the term "thermal ohm" pending selection of a more permanent convention, is an example.

For the thermal "difference of potential" we have well recognized, universally adopted units, the degree Centigrade and degree Fahrenheit, and nothing further needs to be said regarding this quantity.

For the measure of rate of transfer of quantity of heat, corresponding to current in electrical parlance, the literature is not so well off. Instead of a single unit like the ampere, which everyone may be expected to use, we find many in fairly common use. It happens that, because of the extended areas through which heat transfer takes place in so many of the practical problems, it is much more usual to calculate in terms of heat flow per unit area than in terms of total heat flow. Interest centers, thus, on the analog of current density, amperes per unit area, rather than just amperes.

In Table I are collected nine of the units for current density of heat transfer which are in most common use. To assign names to all of these would confuse things far worse than they are at present. It would be absurd to burden a memory with the effort to recall which definition went with which name, and to recite its definition *every* time you use a name is cumbersome. To select some unit or units from the nine, for a christening, is a task which the author prefers to dodge. The advocates for each unit which is tabulated could marshal plausible reasons for not retiring it too far into the background, although probably not all of the combinations present equal claims to precedence as a primary unit. Any committee or individual who essays such selection will have no mean task.

In the metric units, the author inclines very strongly toward emphasis upon the watt as the unit for measuring rate of heat flow, in preference to calories per second or kilocalories per hour. Modern physics is inseparably associated with the concept of heat as identical with energy, and the erg is the natural unit for both if it is for the one. The multiplier, 10^7 (ergs to joules), is invariant, but the multiplier 4.18×10^7 (ergs to calories) rests upon experiment and has to be

TABLE 1.—CONVERSION FACTORS BETWEEN UNITS OF CURRENT DENSITY OF HEAT FLOW. QUANTITY OF HEAT ENERGY TRANSFERRED THROUGH UNIT AREA PER UNIT TIME

		$\frac{watts}{cm^2.}$	$\frac{cal.}{sec. \times cm^2.}$	$\frac{kilo\ cal.}{hr. \times m^2.}$	$\frac{B.t.u.}{sec. \times ft^2.}$	$\frac{B.t.u.}{sec. \times in^2.}$	$\frac{B.t.u.}{hr. \times ft^2.}$	$\frac{B.t.u.}{hr. \times in^2.}$	$\frac{H.P}{ft^2.}$	$\frac{watt}{in^2.}$
$1\ \frac{watt}{cm^2.}$	=	1	0.2391	8606	0.8815	0.006123	3173	22.03	1.246	6.452
$1\ \frac{cal.}{sec. \times cm^2.}$	=	4.183	1	36000	3.687	0.02560	13270	92.17	5.211	26.99
$1\ \frac{kilo\ cal.}{hr. \times m^2.}$	=	$0.0^{3}1162$	$0.0^{3}278$	1	$0.0^{3}1024$	$0.0^{4}7112$	0.3687	0.002560	$0.0^{3}1448$	$0.0^{4}7497$
$1\ \frac{B.t.u.}{sec. \times ft.^2}$	=	1.134	0.2712	9765	1	0.00694	3600	25	1.414	7.320
$1\ \frac{B.t.u.}{sec. \times in.^2}$	=	163.4	39.06	1,406,000	144	1	518,400	3,600	203.6	1,054.1
$1\ \frac{B.t.u.}{hr. \times ft.^2}$	=	$0.0^{3}3152$	$0.0^{3}0754$	2.712	$0.0^{3}2778$	$0.0^{4}1929$	1	0.00694	$0.0^{3}3927$	0.002033
$1\ \frac{B.t.u.}{hr. \times in.^2}$	=	0.04539	0.01085	390.6	0.04000	$0.0^{3}2778$	144	1	0.05654	0.2928
$1\ \frac{H.P.}{ft^2.}$	=	0.8027	0.1919	6908	0.7074	0.004913	2547	17.68	1	5.178
$1\ \frac{watt}{in^2.}$	=	0.1550	0.03705	1334	0.1366	$0.0^{3}9487$	491.8	3.415	0.1931	1

For the table above, the following conversion factors were used:

The calorie was defined to be 4.183 absolute joules. This makes it as closely the 20°C. calorie as present experimental work can tell. The B.t.u. is taken to be 252.00 calories, and is therefore not far from a 70°F. B.t.u. The degree Fahrenheit is 5/9 of a degree Centigrade. The inch is defined to be 1/39.37 of the international meter. Values for "g" and for the pound were used which lead to 1 H.P. = 745.69 watts (absolute). The international electric watt in use at this time (1928) differs from the absolute watt by very closely 1 part in 3000.

redefined every time a more accurate experiment on mechanical equivalent of heat is recognized.

There has always been a considerable confusion between the gram calorie and the kilogram calorie and, for exact work, between 20° calories, 15° calories, mean calories, etc. Although there are standardization movements on foot which are gradually lessening this confusion, the simplest way of reaching international agreement is the gradual discard of *all* calories in favor of the absolute joule, regarding which no differences of definition can arise. Most calorimeters are standardized today primarily by electric heating, so that their basis of reference is really the electric joule, and not the calorie at all.

For heat transfer work let us adopt, in the metric system, the watt per cm². as the primary unit for heat flow per unit area, in other words, the watt as the unit of rate of heat transfer, the "current." As this unit already has a name, no new proposal is involved in regard to naming it.

The unit of thermal resistance in the metric system will then be one degree Centigrade per watt, which needs a name. The physicist whose name is always associated with the subject of heat conduction is Fourier, and if his name is to be given to any unit it would seem most appropriate that this unit be one of those fundamentally involved in the transfer of heat by conduction. The logical one is the metric unit of thermal resistance.

The *fourier* will thus be visualized as that resistance between two isolated surfaces, one degree Centigrade different in temperature, which makes the rate of heat transfer one watt (10^7 ergs per second). It happens that the thermal properties of an average brass at room temperatures are such as to give us a concrete model in terms of a slab 1 cm. thick. Such a slab with a temperature difference of 1°C. between the faces has a heat flow of 1 watt per cm². of area, so that its resistance is 1 fourier for each cm². of cross section.

Silver and copper have approximately four times the thermal conductivity of brass, or one-fourth the resistivity, so that we picture the fourier in terms of these metals by noting that a slab of 1 cm. thickness has a resistance per cm². cross section of about 1/4 of a fourier, or a slab 4 cm. thick secures a resistance of 1 fourier per cm².

To make a laboratory fourier, we might cut a right prism of copper 1 cm². in cross section (the prism may be circular, square, or any other shape) and about 4 cm. long. The model is just as definite as is (approximately) 60 meters of copper wire 1 mm². in cross-section for

an electrical ohm. The difference as regards laboratory use of the two pieces of apparatus is, of course, that the wire ohm requires few precautions to keep electric current from "leaking off the sides," while the metal fourier needs some auxiliaries to keep the heat flow axial.

Just as we might make our ohm (in wire of 1 square millimeter section) from either 60 meters of copper or about 1 meter of nichrome alloy so we may visualize our fourier in a variety of ways. In prisms of 1 square centimeter area, it is 4 cm. length of copper, or 1 cm. of brass, or about 0.1 mm. of glass, a few microns only of asbestos, or of cork or of air.

A few approximate values of thermal resistivities are collected in Table II to illustrate relative orders of magnitude:

It may be noted in passing that while resistivity of a material involves linear dimensions of a unit specimen thereof, resistance does not involve these (except insofar as a unit of distance is inherently a part of the definition of an erg). That is to say, a fourier is defined when the degree Centigrade and the watt have been specified, and the user of the unit may, if he likes, combine it with feet or inches instead of with centimeters, in any computation. The unit should therefore prove acceptable not only to physicists and chemists, but also to the large class of engineers, especially electrical engineers, who are accustomed to watts and degrees Centigrade. It will obviously not be acceptable to those engineers who prefer British thermal units to watts and Fahrenheit degrees to Centigrade. Instead of the fourier, the mechanical engineers engaged upon refrigeration problems, boiler insulation, etc., will want a unit of thermal resistance based on British measures. The difficulties which will be encountered in selecting one have been hinted above, in the section devoted to heat current-density.

Common practice among heat insulation engineers in America today is to base heat transfer computations on the use of a dimensional unit which is the board foot of the lumber industry. Dimensions in two coördinates are taken in feet, and the third coördinate in inches. This "commercial conductivity" as it has been named (B.t.u. per hr. per sq. ft. for rate of heat flow, and °F. per inch for temperature gradient), while very useful as a secondary unit for computation in many practical applications, is clearly unsuited to give a *primary* unit of resistivity. Any system which measures distances east and west in inches and those north and south in feet, is fraught with pitfalls. Imagine the mental gymnastics for a measurement northeast and southwest. In other words, the moment we attack problems where thermal resistivity is to be combined with viscosity or velocity or any other physical prop-

TABLE II.—THERMAL RESISTIVITIES AT 20°C. EXPRESSED IN FOURIERS FOR A CENTIMETER CUBE

Silver	0.239
Copper	0.258
Aluminum	0.49
Brass (30% Zn)	0.93
Iron	1.6
Nickel	1.7
Steel (1% C)	2.1
Constantan	4.4
Mercury	12.0
[Ice at 0°C.]	45
Glass*	133
Concrete*	140
Water	170
Mica* (⊥ laminations)	200
Firebrick*	200
[Firebrick 25°C. to 1000°C.]	90
Brick masonry*	250
Leather*	600
Hydrogen	600
Hard rubber	610
Helium	690
Rubber* (over 90%)	700
Wood (Virginia pine across grain)	710
Paper*	1000
Asbestos* (wool)	1100
Cork*	2000
Cotton batting (loose)	2500
Wool (loose)	2500
Air	4100
Carbon dioxide	6700

* Substances marked with the asterisk vary widely in thermal conductivity according to composition. For limits of such variation, consult International Critical Tables, Vol. II. The figure listed above for any such material represents the author's estimate of the "best guess" for use in those cases where the composition of the material is not specified.

In preparing this table, the author has consulted Vol. II, I. C. T. and has courteously been furnished advance values for some other materials by the editors of I. C. T. For still other materials, grateful acknowledgment is made to the staff of the U. S. Bureau of Standards, for advice in selecting most probable values in the light of present information.

erty, we are obliged to have a unit which is self-consistent, feet throughout or inches throughout, but not both at once. The board foot as a geometrical unit for resistivity can never be more than a secondary choice used for a particular class of problems.

Dropping the geometrical factor of resistivity and considering a unit for resistance, we note that commercial conductivity is expressed in B.t.u. per hour, while the second is the universally accepted unit of time. This means that although a resistance unit based on the hour would contain no inherent inconsistency, constant vigilance would

have to be exercised with respect to using a conversion multiplier if it were brought into a formula with other physical quantities, for example "g," which would surely be in seconds, not hours.

The logical primary unit for thermal resistance in British measure is of course based on the British thermal unit, second, foot and Fahrenheit degree. The argument against dragging such a unit into the foreground, is the very cogent one that aside from a very few papers on theory of heat transfer, nobody would use it. The simple fact is that this combination has not found favor in calculations pertaining to engineering structures.

All in all, the author is inclined to beg the question of selecting and naming a unit of thermal resistance in British measure. The multiplicity of usage is too discouraging. In deference to this lack of standardization, would it not be quite as well to promote the universal tabulation of resistivities (and reciprocally, conductivities) in fouriers for the centimeter cube, and suggest to each engineer that he write in his handbook at the margin of the table, the multiplying factor which converts the tabulation in fouriers to that *particular combination* of British measure elements which his past experience has led him to prefer for his thermal conduction calculations.

The fourier is proposed as the name for a thermal resistance such that each °C. of temperature head applied at its terminal surfaces gives a heat flow of 1 watt (10^7 ergs per second).

APPENDIX

Factors by which to multiply a value of thermal resistivity, expressed in fouriers for a centimeter cube, to get values in other systems of units

Heat flow in	*Temperature Gradient*	*Multiplier*
Watts per cm².	°C. per cm.	1
$\frac{\text{Calories}}{\text{sec.}}$ per cm².	°C. per cm.	4.18
$\frac{\text{Kilocalories}}{\text{hr.}}$ per m².	°C. per m.	0.0116
$\frac{\text{B.t.u.}}{\text{hr.}}$ per ft².	°F. per ft.	.0172
	°F. per inch	.00143
	°C. per inch	.00080
$\frac{\text{B.t.u.}}{\text{hr.}}$ per in².	°F. per inch	.206
	°C. per inch	.115
$\frac{\text{B.t.u.}}{\text{sec.}}$ per in².	°F. per inch	745
	°C. per inch	414
H.P. per ft².	°F. per foot	44
	°F. per inch	3.67
Watts per in².	°C. per inch	0.394

PHYSICAL GEOGRAPHY.—*A preliminary note on blue-green algal marl in southern Florida in relation to the problem of coastal subsidence.*[1] ALFRED P. DACHNOWSKI-STOKES, U. S. Bureau of Chemistry and Soils, and R. V. ALLISON, Florida Agricultural Experiment Station.

In 1922 a paper was presented before the National Academy of Sciences in which the broad outlines of the stratigraphic successions between American and European peat deposits were pointed out, and a correlation was attempted of glacial, climatic and life stages since the last Ice-age.

In a later paper, dealing with the profiles of peatlands in New England, which appeared in 1926 in *Ecology*, evidence was set forth showing that the Atlantic Coast of North America had suffered a geologically recent subsidence or a rise of sea-level. The observations offered in the present paper are believed to demonstrate coastal stability during the last few thousand years.

Wide expanses of the bedrock floor of limestone in the southern part of the Florida Peninsula are covered by gray marl. The material varies in thickness from 1 to 2 feet and on it are found mangrove islands, saw-grass marshes, prairie vegetation north of Flamingo, and portions of the great cypress swamps to the west of the Everglades. In origin, appearance, and manner of deposition, this marl is unlike that representing chemical precipitation, the aggregation of shells, fragments of chara, or the calcareous ooze produced by bacterial action. In the present case blue-green algae constitute the bulk of the organisms which are building up and extending to a surprising degree the marly soil of the lower glades and of the shores along the southern mainland.

Notwithstanding their great economic interest to man as agents of decay, fermentation, and disease, the aggregate work accomplished by these organisms is far from being understood. Until recently bacteria and blue-green algae received little attention as agencies that encrust themselves with lime carbonate and thus stand out as builders of extensive areas of mineral soil. Today there is evidence to show that precipitation of calcium, silicon, iron, sulphur, and part of the colloidal organic material in soils is caused by algae and bacteria. From the geologist's standpoint Glock (4) and Diener (2) have reviewed the bibliography establishing these forms as reliable fossils of stratigraphic value and for the interpretation of past climatic changes;

[1] Received July 30, 1928.

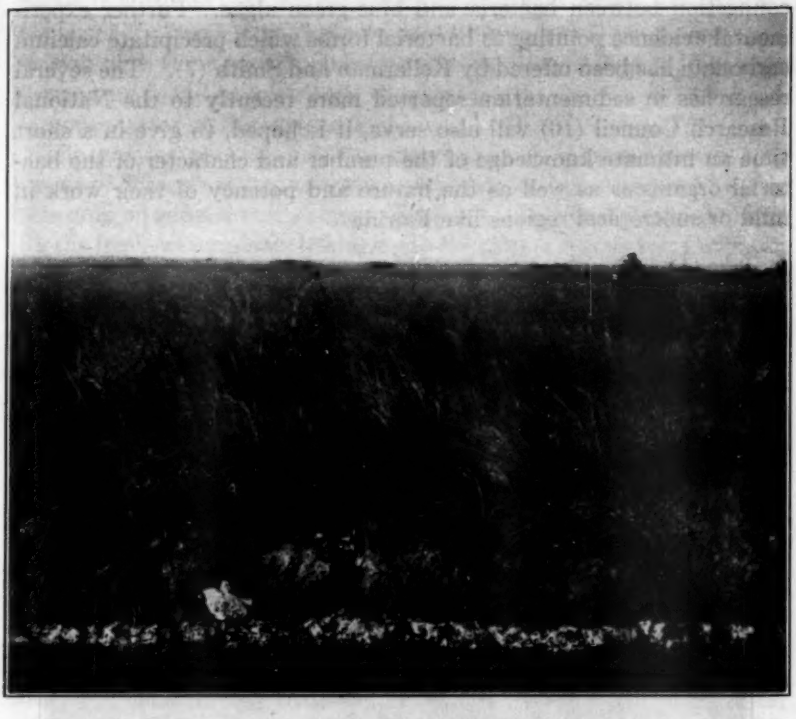

Fig. 1.—Profile features of gray marl along the Ingraham Highway between 10 and 20 miles north of Flamingo, Florida. Photographed February 22, 1928, by R. V. Allison.

the organisms have been recorded since early geologic times, and are reported as existing in peat deposits. Botanists and ecologists, on the other hand, are beginning to direct attention to the lower order of plants as indicators, whose habitat includes extremes of temperature, geographic conditions, and aerial situations.

Investigations and views concerning the active organisms, the process, and the secondary changes accompanying precipitation of calcium carbonate have not, as yet, thrown much light on the formation of marl under natural field conditions. In regard to bacterial organisms Drew (3) and Vaughan (11) have discussed in considerable detail the accumulations of calcareous sediments and oolite made by the action of denitrifying bacteria in shallow water adjacent to the keys and living coral reefs of Florida. The specific identification of the living forms has been attended with difficulty, owing to the close

connection between bacteria and blue-green algae. Further experimental evidence pointing to bacterial forms which precipitate calcium carbonate has been offered by Kellerman and Smith (7). The several researches in sedimentation reported more recently to the National Research Council (10) will also serve, it is hoped, to give in a short time an intimate knowledge of the number and character of the bacterial organisms as well as the nature and potency of their work in mild or subtropical regions like Florida.

Fig. 2.—Profile section in dense growth of *Distichlis spicata*, about 6 miles north of Flamingo, Florida, along southern extension of Dixie Highway. Photographed February 22, 1928, by R. V. Allison.

With reference to blue-green algae there is apparently a great lack of information regarding their number and geographical distribution in the southern part of the United States. Tilden (9) records the specific description of only 31 filamentous blue-green algae so far known to exist in Florida. They are grouped chiefly among 5 families and 19 genera.

Some of the material described below was collected in the Ever-

glades in 1919 and kept dry for subsequent study. It was not until the winter of 1927 and 1928, during a reconnaissance trip made in coöperation with the Agricultural Experiment Station of Florida, that algal forms were found to contribute so materially to the deposition of calcareous matter at the surface of the soils.

The greater part of the blue-green algae has been identified by means of Tilden's volume on Myxophyceae (9), but they are classified here only on general resemblances of form.[2] The most abundant algae are the fresh-water plants belonging to the genera *Scytonema*, *Calothrix*, *Lyngbya*, and *Dichothrix*. The latter, together with others, occurs as a thin, soft, greenish-blue matted coating, closely attached to the friable, calcareous material about 1 or 2 inches below the surface. They form gray, laminated or flaky and nodular incrustations, which present a cavernous structure to a depth varying from 5 to 8 inches. Underneath is grayish-white, compacted, harder marl, more or less amorphous, plastic when wet, and frequently dark gray in color and mottled from rootlets to the contact line with the underlying bedrock limestone. Plate 1 shows a profile exposure at type localities along the Ingraham Highway between 10 and 20 miles north of Flamingo.

The vegetation of these places is similar in aspect to that of other parts of southern Florida, but it differs in composition. The trees and shrubs are woody evergreens. Among the herbaceous plants commonly observed on the "Ingraham marl" are sedges and grasses, (*Cladium mariscus*, *Eleocharis cellulosa*, *Rhynchospora tracyi*, *Dichromena colorata*, *Spartina bakeri*), reeds, and species of *Aletris*, *Crinum*, *Flaveria*, *Ludwigia*, and *Sisyrinchium*. The principal types of vegetation native to this region have been described from diverging viewpoints by Harshberger (6) and by Harper (5) whose paper contains many bibliographic references.

Very little use has been made thus far of any of this type of soil, but some of it has recently been cultivated for early tomatoes.

There are no detailed soil surveys for southern Florida, and hence the distribution and acreage of this material are practically unknown. The marls described by Matson and Sanford (8) in their paper on the geology and ground waters of Florida extend over hundreds of square miles, but too little is known to map them as a soil type. The northern boundary of the gray algal marls appears to coincide with a line near the Tamiami trail between Miami and Collier County. South of this line the marl was observed along the Ingraham Highway between

[2] A more detailed identification is being made in connection with precipitation studies to be published later.

Homestead and Flamingo, merging into the limy ooze of the islands along the shores of the southern mainland.

The average rapidity with which algal marl is now accumulating is not known, nor the rate of increase under favorable and unfavorable circumstances. The thickness is nowhere great and is only moderately irregular, in part due to the uneven surface upon which the marl was deposited at a particular place. From the profile features of the layer it may be inferred that throughout the time following the emergence of southern Florida from the sea, the conditions governing the deposition of the calcareous material were not marked by great disturbances. The emergence of the "lower Glades" and mainland with respect to sea level has been small and began probably only a few thousand (perhaps 3000 to 4000) years ago. In comparison with the series of profiles obtained in the "upper Glades" and on Torrey Island in Lake Okeechobee, already noted in another connection (1), the evidence discovered seems to prove essential coastal stability during the past few thousand years. The algal marl layer was formed during relatively recent or late Pleistocene times, and since then there has been no appreciable change in the relative positions of land and sea in southern Florida. A fuller treatment of this question is reserved for a later paper.

LITERATURE CITED

(1) DACHNOWSKI, A. P., *The correlation of time units and climatic changes in peat deposits of the United States and Europe.* Proc. Nat. Acad. Sci. **8**: 225–231, 1922.

(2) DIENER, C., *Grundzüge der Biostratigraphie.* Leipzig, 1925.

(3) DREW, G. H., *On the precipitation of calcium carbonate in the sea by marine bacteria.* Tortugas Lab. Carnegie Inst. Washington, **5**: 1914.

(4) GLOCK, W. S., *Algae as limestone makers and climatic indicators.* Amer. Journ. Sci. Ser. V. **6**: 377–408, 1923.

(5) HARPER, R. M., *Natural resources of Southern Florida.* Eighteenth Ann. Rep. Fla. State Geol. Surv. 27–206, 1927.

(6) HARSHBERGER, J. W., *The vegetation of South Florida, south of 27′ 30° north, exclusive of the Florida Keys.* Trans. Wagner Free Inst. Sci. **7**: 49–189, 1914.

(7) KELLERMAN, K. F., and SMITH, N. R., *Bacterial precipitation of calcium carbonate.* This JOURNAL **4**: 400–402, 1914.

(8) MATSON, G. C. and SANFORD, S., *Geology and ground waters of Florida.* U. S. Geol. Survey, Water Supply Paper 319, 1913.

(9) TILDEN, J., *Minnesota algae.* I. *The Myxophyceae of North America and adjacent regions.* Bot. ser. 8, 1910.

(10) TWENHOFEL, H. H., *Researches in sedimentation.* National Research Council, Washington D. C., 1927.

(11) VAUGHAN, T. W., *Remarks on the geology of the Bahama Islands and on the formation of the Floridan and Bahaman oolites.* This JOURNAL **3**: 302–304, 1913.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

THE ACADEMY

218TH MEETING

The 218th meeting was a joint meeting with the Philosophical Society and was held in the Assembly Hall of the Cosmos Club on the evening of Thursday, January 19th. The general subject of discussion was methods of election, including the election of officers and governing bodies of Scientific Societies.

Program: L. B. TUCKERMAN of the Bureau of Standards: *Theoretical principles underlying balloting.* The complete expression of the will of an electorate on a plurality of candidates for the same office or plurality of similar offices would be contained in a single preferential vote if all questions of expediency could be eliminated from the vote. Certain mathematical conditions must be imposed on the count to ensure the complete elimination of expediency from the vote.

These conditions are not realized in any count so far devised, and may be impossible of realization. The mathematical problem, being a problem in combinatory analysis, is difficult. A number of able mathematicians have worked on the problem with the result that it is known that in certain broad classes of counts expediency can not be wholly eliminated from the vote. Further investigation is desirable.

Although expediency may never with certainty be eliminated from the vote, two different counts have been devised, meeting two distinct needs, in which expediency in voting has been so nearly eliminated as to be negligible from a practical standpoint. These are the Hare count for electing a representative body and the Condorcet count for filling a single office. These counts of a preferential ballot are far superior to any other known methods of election, and may be considered a practical solution of the problem of securing an effective expression of the will of an electorate. (*Author's abstract.*)

GEORGE H. HALLETT, JR., Secretary of the Proportional Representation League: *An appraisal of election methods.* Dr. Tuckerman has suggested as a criterion for a perfect method of election the requirement that it shall always give most effect to a person's vote if he expresses on the ballot his real wishes. This seems a reasonable requirement, but unfortunately it conflicts with another which seems just as reasonable and perhaps even more important. It can be proved that a method of election under which it would never be profitable for the voter to falsify his real wishes would sometimes defeat a candidate for a single office who was preferred to all other candidates by an absolute majority of the voters.[1] Under such circumstances the unsolved question whether Dr. Tuckerman's test can be satisfied at all is one of considerable academic interest but probably not of great practical importance.

This does not mean that the test has no value. It can and should be satisfied for most practical purposes without sacrificing anything else of more importance.

The most workable system so far proposed seems to be the Hare system of proportional representation. This system is not mathematically perfect.

[1] The proof is given in footnote 14 on pages 396–397 of *Proportional Representation* by HOAG and HALLETT (Macmillan).

It defeats arbitrarily the candidate who stands lowest on the poll and occasionally such a candidate might be preferred to each other candidate considered singly by a majority of the voters. It is possible to overcome this defect also by introducing complications in the rules for counting,[2] but the Hare system in its usual simple form offers a small probability of error and its political as well as theoretical advantages over our ordinary methods are very great. It is in use for the election of important single officers in Australia and in the provinces of Manitoba and Alberta.

When a group of officers are being elected together to a legislative body, it is possible to do still better. For when a single place is to be filled it is only possible at the best to satisfy one point of view, but when a number of places for the same office are to be filled together it is possible to give just representation to all important points of view.

On this question of representation there is a great deal of confusion in the public mind. It is quite generally assumed that the principle of majority rule demands that each particular member be a majority choice. This is frequently qualified by a division of the whole territory to be represented into districts, in some of which a minority party may have a local majority and so get representation, but the idea of assuring representation to minorities whether they are a majority in any one neighborhood or not is thought to conflict with majority rule.

In fact, however, there can be no assured majority rule in a representative body without a representation of all elements in proportion to their voting strength.

The so-called majority methods of election very frequently defeat the majority will. In the last two Congressional elections in the State of New York the Republicans have polled a majority of the votes for Representatives and the Democrats have elected a majority of the members. This was because the Republican minorities in Democratic districts (like the Democratic minorities in Republican districts) were unrepresented. In 1888, when Harrison was elected President over Cleveland, Cleveland had 100,000 more popular votes. A similar miscarriage of justice happened on two other occasions. This was because the minorities in each state were unrepresented in the electoral college.

Though minority rule does not usually stand out so glaringly on the face of the election returns, it is in fact the rule rather than the exception. The remarkable phenomenon of machine rule the country over, with a majority of the people almost everywhere opposed to machine rule, is due simply to the fact that we do not provide a means of representation for the minorities into which the anti-machine majority is divided.

This situation can easily be remedied by a change in election methods to give representation to all elements in proportion to their voting strength. Three things are needed: first, districts large enough to elect several members each; second, a single vote for each voter instead of as many votes as there are members to be elected in the district, so that the largest group cannot elect its entire slate to the exclusion of others; and third, a vote that is transferable, so that, if the voter's first choice cannot be elected by it, it will not be wasted but be transferred to his second choice or third.

This combination of principles gives us the Hare system of proportional representation, otherwise known as the single transferable vote. In its usual forms it is not mathematically perfect and contains various minor

[2] HOAG and HALLETT, *Proportional Representation*, pp. 494 ff.

defects which could be remedied by the introduction of considerable complications in the counting rules. Such complications, however, would have little effect on the results secured and without them the Hare system offers a practical remedy for most of our political ills which are not inherent in the people themselves. It is used for city elections in Cleveland, where it has secured the services of such able independent councilmen as Professor A. R. Hatton, charter consultant of the National Municipal League; in Cincinnati, where it immediately deposed one of the most thoroughly entrenched political machines in the country and changed the city from one of the worst-governed to one of the best-governed to be found anywhere; in Ashtabula and Hamilton, Ohio, and Boulder, Colorado, and in many cities abroad. In other countries it has been inaugurated for provincial elections, as in Manitoba, Alberta, and Tasmania; and even in national elections, as in the Irish Free State, and in Great Britain for a few university members of the House of Commons. It requires no primaries and in one election gives a free and practically complete expression of the real wishes of the voting public. (*Condensed from author's manuscript.*)

A demonstration election was then given with the audience as voters. The votes were counted by members of the audience under Mr. Hallett's direction and the results put on a blackboard and explained.

219TH MEETING

The 219th meeting was held in the Assembly Hall of the Cosmos Club on the evening of Thursday, February 16th, 1928.

Program: DAYTON C. MILLER, Professor of Physics in the Case School of Applied Science: *Photographing and analyzing sound waves.* The address was illustrated by latern slides and by experiments, including experiments with the "phonodeik," an apparatus devised by Dr. Miller for projecting "living" waves directly onto a screen.

220TH MEETING

The 220th meeting was held in the Auditorium of the National Museum on the evening of Thursday, March 15th, 1928.

Program: Showing of a moving picture film entitled *The Mechanics of the Brain,* prepared by Prof. IVAN P. PAVLOV, Director of the Physiological Laboratories in the Russian Academy of Sciences. The picture presented a series of experiments on children and animals chosen to illustrate the mechanism of the reactions to various external stimuli. Especial attention was given to the development of "conditioned reflexes," a subject that has been especially studied by Pavlov and his collaborators. The film was exhibited through the courtesy of the American Society of Cultural Relations with Russia.

221ST MEETING

The 221st meeting was held in the Auditorium of the National Museum on the evening of Wednesday, May 16th, 1928.

After calling the meeting to order the President of the Academy announced that a proposed revision of the By-Laws of the Academy, intended to clarify doubtful points and to meet changing conditions, had been duly proposed by three members of the Academy. Under the provisions for amending the By-Laws these proposals were automatically referred to the Board of Managers

for further consideration and for submission to the Academy at a future meeting.[3]

Program: FRIDTJOF NANSEN, Arctic explorer, professor of Oceanography at the University of Oslo and President of the International Society for the Exploration of the Arctic by means of the Airship: *Problems of Arctic exploration.* The speaker emphasized the importance of polar exploration as against the popular view that polar expeditions are quests for adventure or bids for notoriety. He especially emphasized the importance of a knowledge of meteorological conditions in the polar regions and compared a study of meteorology with the polar regions omitted to a study of the steam engine with the condenser omitted. Many modern developments of meteorology are based on the "polar-front" theory.

The speaker then described the methods of polar exploration as they were in the nineties when he and his companions drifted across the polar sea in the *Fram* and told of some of the adventures encountered in that memorable expedition. He then described the advantages of exploration by airship and outlined the plans for a proposed expedition. At the conclusion of the address the audience gave the speaker a rising vote of thanks.

WALTER D. LAMBERT, *Recording Secretary.*

[3] The proposed changes are in fact the work of a special committee of the Board of Managers and have been considered by the Board itself at several meetings and approved in principle. Requests for further information regarding these changes may be addressed to the Corresponding Secretary, who is also Chairman of the special committee.